

# Fairness in Combinatorial Auctions

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## Abstract

The market economy deals with many interacting agents such as buyers and sellers who are autonomous intelligent agents pursuing their own interests. One such multi-agent system (MAS) that plays an important role in auctions is the combinatorial auctioning system (CAS). We use this framework to define our concept of fairness in terms of what we call as “basic fairness” and “extended fairness.” The assumptions of quasilinear preferences and dominant strategies are taken into consideration while explaining fairness. We give an algorithm to ensure fairness in a CAS using a Generalized Vickrey Auction (GVA). We use an algorithm of Sandholm to achieve optimality. Basic and extended fairness are then analyzed according to the dominant strategy solution concept.

**Keywords:** fairness, optimality, multi-agent systems, combinatorial auctions, mechanism design

## 1. Introduction

The term “auction” refers to a mechanism of allocating single or multiple resources to one or more agents (or bidders) (Conitzer, 2009). In recent years, computer scientists, rather than just economists, are interested in auctions. The increase in computing power and improved algorithms have paved the way for combinatorial auctions. Here multiple items are for sale by the auctioneer and bidders can bid for a bundle of items (also called packages). In a multi-agent system (MAS), we consider these bidders and the auctioneer as autonomous agents who act in a self-interested manner in their dealings with one another. Similarly, even in MAS dealing with resource allocation other than by auction, there are self-interested autonomous agents (Bredin et al., 2000; Sycara, 1998). We study a framework where optimality is a desirable property but fairness is a required property. An excellent example of such a framework is a combinatorial auctioning system (CAS) where the two

most important issues pertaining to resource allocation are optimality and fairness. A CAS is a kind of MAS whereby the bidders can express preferences over combinations of items (Nisan, 2000; Narahari & Dayama, 2005).

We assume in this paper that an agent’s valuation of an item does not change based on other agents’ private information (i.e., some evidence which affects the valuation of an agent), that utilities are quasilinear (i.e., utility is linear in terms of money), and that there are no externalities (i.e., an agent that does not win an item neither cares which other agent wins it, nor worries about how much other agents pay for it) (Conitzer, 2009). This is realistic, as seen for example in relation to the Nigerian Communications Commission auction described below, where, “The decision to charge bidders what they bid was accepted by bidders and observers as fair and transparent despite the difference in some of the payments for identical licenses.” (Koboldt, Maldoom, & Marsden, 2003, p. 30) In such scenarios, each agent holds different preferences over the various possible allocations and hence concepts like individual rationality, fairness, optimality, efficiency, etc., are important (Chevaleyre et al., 2006).

We introduce the concept of fairness in the auction mechanism. Although the notion of fairness is of course well known in general, it does not seem to have been clearly defined with respect to auction processes in particular. We propose two types of fairness, namely *basic fairness* and *extended fairness*. We explain basic fairness using the concept of equitable distribution along with the respective preferences. Extended fairness is explained such that envy-freeness prevails in the allocation and the entire resource is allocated to the winning bidder. We introduce a *fairness table* consisting of fair values as perceived by bidders and auctioneer; this is sealed at the start of the bidding process. We give emphasis to fairness, unlike the classical approach where revenue maximization is the only goal required in auctions. To achieve fairness, the proposed algorithm explains a novel payment scheme which is applied at the end of the bidding process, where we determine the final amount payable to the auctioneer by the winning bidder. We ensure that this process is considered to be fair by both bidders and the auctioneer by means of extended fairness. We handle the special case of a tie in the bidding process using equitable distribution, and ensure that basic fairness is achieved. The mathematical formulations of fairness concepts in combinatorial auctions are explained, and a detailed analysis is presented to highlight some of the properties exhibited by our payment scheme.

In our mechanism, there are self-interested bidders and an auctioneer, who express their perceptions of the fair value of the resources through a data structure called the fairness table. Here an auctioneer acts as a facilitator to ensure that an item achieves its fair value. We consider optimality as the desired property and fairness as the required property. We illustrate this using a combinatorial auction framework, in which multiple items are simultaneously up for sale and a bidder can bid for any bundle of items. The optimal allocation of resources is discussed using an algorithm of Sandholm where we obtain the winning bidders. The incentives to the winning bidders are provided through the Generalized Vickrey Auction (GVA) and Algorithm 1. We apply the fairness concept using the fairness table and Algorithm 1.

The auctioning of the electromagnetic spectrum is one of the well-known applications of combinatorial auctions. The first-ever combinatorial auctioning of the radio spectrum

was held in Nigeria in 2002 (Koboldt et al., 2003). A single-round sealed-bid combinatorial auction (not the Simultaneous Multiple Round Auctions (SMRA) used in other countries) was conducted for regional fixed wireless access (FWA) licenses; the decision for this format was made as SMRA was impractical in Nigeria due to its insufficient communication infrastructure, and as the NCC did not wish to have a lengthy auction. Some 67 out of the 80 licenses available were allocated, with successful bids amounting to 3.78 billion naira (38 million US dollars). Here, the complementarity and substitutability of licenses were the important factors for choosing a combinatorial auction. The cost of the allocation process was an important factor, and the Nigerian Communications Commission (NCC) did not want to discourage smaller bidders, with its primary goal being efficiency and transparency.

We find in the case of the combinatorial auction conducted by NCC the following problem: “The final choice of auction design rested heavily on information revealed about the regional structure of demand from initial applications. It was therefore critical that the application process created incentives for bidders to reveal such information.” (Koboldt et al., 2003, p. 24) This problem is taken care of in our approach as we introduce the fairness table which is to be populated at the beginning of the auctioning process. This table allows us to see the regional structure of demands through the fair values assigned to the resources. We provide higher rewards for bidders who truthfully give their fair values, by Theorems 5.1 and 5.2.

It was also observed by the NCC that bidders defaulted on their winning bids in a significant number of cases, though not enough to undermine the overall process (Koboldt et al., 2003). We can decrease the number of bidders defaulting provided we satisfy Theorem 5.2. This makes it less rewarding for bidders to bid way beyond their capacity, which in turn decreases the possibility of winning bidders defaulting.

Since the importance here is on transparent and fair allocation, we can apply our method to ensure fairness in combinatorial auctions. We start with introducing some of the related work in Section 2. Next, we explain different notions of fairness with formal definitions in Section 3. This is followed by our study of CAS in Section 4; Algorithm 1 in Section 4.4 is used to extend the payment scheme to achieve fairness in CAS with an example. Section 5 gives a detailed analysis of fairness using mechanism design under quasilinear settings (Shoham & Leyton-Brown, 2009). We conclude with Section 6, which offers some conclusions about our efforts, and some suggestions for further work along these lines.

## 2. Related Work

In this section, we review different definitions of fairness as they have been proposed in the multi-agent literature. The problem of fair allocation is being resolved in various MAS by using different procedures, depending upon the technique of allocation of goods and the nature of goods. Its welfare implications in different systems were explored by Rabin (Rabin, 1993). Brams and Taylor give an analysis of procedures for allocating divisible and indivisible items and for resolving disputes among the self-interested agents (Brams & Taylor, 1996). One of the procedures described by them is the “divide and choose” method of allocation of divisible goods among two agents to ensure the fair allocation of goods which also

exhibits the property of “envy-freeness,” a property first introduced by Foley (Foley, 1967). Lucas’ method of markers, and Knaster’s method of sealed bids are described for MAS comprising more than two players and for the division of indivisible items. The adjusted-winner (AW) procedure is also defined by Brams (Brams, 2005) for envy-freeness and equitability in two-agent systems. Various other procedures like moving-knife procedures for cake cutting are defined for the MAS comprising three or more agents (Brams, 2005; Barbanel & Brams, 2004).

The auction mechanism proposed by Biggart (Biggart, 2002) provides an economic sociology perspective. There, fairness can mean different things for bidders and auctioneer. The auctioneer may consider a process fair which in fact only gives him the maximum revenue, whereas the bidders may consider a process fair which only gives the auctioneer the least return on all items. The most important consideration overall is to sustain the community’s faith in the fairness of the process. This does not mean that buyers and sellers cannot press their advantage, but they are allowed to do so only insofar as the community as a whole considers their actions appropriate and acceptable.

A concept of verifiable fairness in Internet auctions has been proposed by Liao and Hwang (Liao & Hwang, 2001). This was to promote trust in Internet auctions. The scheme proposed provides evidence regarding policies implemented so that the confidence of bidders increases and they consider it to be fair. Most of these auctions see transparency in the auctioning process and rules as the basis for ensuring fairness in the system, but clarity regarding fairness still remains wanting.

The Nash bargaining concept is used by many economists. In Nash bargaining, there is no particular winner against a bargain. If the amount requested is within the total amount available at the owner then they get their share, but if the demand is more then they get nothing. In our case, this is not the case as we have a winner in all circumstances even if the auctioneer is facing a loss. Also the extended fairness concept is not present in Nash bargaining to acquire a desired product whereas in our case bidders pay a price to achieve extended fairness.

The game-theoretic concept of Shapley value (Shapley, 1953) describes the fairest allocation of collectively-gained profits between several collaborative agents. This is one approach used in coalitional games. Though this deals with fair allocation, it is restricted to a mechanism where the actors contribute in a coalition. The profits obtained are allocated in a fair manner. The Shapley value is different from our approach, as we do not take into account prior understanding or coalition among the bidding agents in our discussion.

Fairness as a collective measure has been considered by Moulin (Moulin, 2004), who proposes aggregate or collective welfare which is measured in terms of an objective standard or index that assumes equivalence between this measure and a particular mix of economic and non-economic goods which gives happiness to a varying set of individual utility functions. This tries to capture social welfare and commonwealth to be incorporated into every individuals’ happiness equations. Though debatable, it provides an excellent introduction to the concept of fairness.

A Distributed Combinatorial Auctioning System (DCAS) consisting of auctioneers and bidders who communicate by message passing has been proposed (Rasheedi, Chatterjee, &

Rao, 2009). Their work uses a fair division algorithm that is based on DCAS concept and model. It also discusses how basic and extended fairness implementations may be achieved in distributed resource allocation.

The fair package assignment model proposed by Lahaie and Parkes (Lahaie & Parkes, 2009) is defined on items having pure complements or super additive valuations. This model does not address combinatorial package assignments which involve both complements and substitutes in general. Their model provides fairness to a “core” which contains a set of all distributions which are considered competitive—no fairness is posited for other distributions. Hence the bidders whose distributions lie outside the core do not get the benefits of fair assessment. In the case of multiple-round combinatorial auctions, for example, bidders whose bids are not in the core during earlier rounds are not in contention in later ones. This scheme seems unfair in a fundamental way, as it effectively discriminates against bidders who cannot make it into the core. In our model only truthfulness in bidding is considered, and no bidders are distinguished based on whether their bids lie inside or outside a putative core.

However, the term *fairness* is defined differently in various MAS with regard to the resource allocation. In some MAS, it can be defined as equitable distribution of resources such that each recipient believes that he receives his fair share. Thus, no agent wants somebody else’s share more than its own share; such division is therefore also known as envy-free division of resources (Brams, 2005). Thus fair allocation is achieved if it is efficient, envy-free and equitable (Brams & Taylor, 1996).

### 3. Definitions of Fairness

Our additional notions of fairness in various MAS are basic fairness and extended fairness. This section defines the various notions about fairness in combinatorial auctions in a MAS.

#### 3.1 Terminology

Let our CAS be a MAS which is defined by the following entities:

1. The total number of resources is represented by  $m$  and the total number of bidders by  $n$ .
2. The set  $R = \{r_0, r_1, r_2, \dots, r_{m-1}\}$  is a set of  $m$  resources  $r_i$ , and  $2^R$  denotes the power set of  $R$ .
3. The set  $B = \{b_0, b_1, b_2, \dots, b_{n-1}\}$  is a set of  $n$  bidders  $b_j$ .
4.  $a$  is the auctioneer who initially owns all the resources.
5. A package  $S$  is some subset of the set of resources, i.e.,  $S \subseteq 2^R$ .
6.  $\mathbb{R}$  is the set of real numbers.

For instance, consider a CAS that comprises three bidders  $b_0, b_1, b_2$ , an auctioneer denoted as  $a$ , and three resources  $r_0, r_1, r_2$ . Each bidder is privileged to bid upon any combination of these resources. We denote the combinations or subsets of these resources as  $\{r_0\}, \{r_1\}, \{r_2\}, \{r_0, r_1\}, \{r_0, r_2\}, \{r_1, r_2\}, \{r_0, r_1, r_2\}$ .

**Example 3.1.** A package for a bidder winning the subsets  $\{r_0\}$  and  $\{r_1\}$  is defined as  $\{\{r_0\}, \{r_1\}\}$ .

We also consider the concept of weight while assigning the fair value. Here, weight is not the physical weight but is used as a multiplicative factor for describing the desirability of the package by the bidder. If a higher weight is assigned to a package, then it will result in a higher fair value. This expresses the well-known fact that a bidder is likely to assign a higher fair value to a resource that is desired or needed than to one that is not, even when the two resources have the same intrinsic value (e.g., a starving man is likely to assign a far higher value to a meal than to any other commodity of equivalent intrinsic worth).

**Definition 3.2.** Let us define some important terms used in our later discussion, as follows:

1. The *initial value* of an item is defined as  $\Omega : B \times R \rightarrow \mathbb{R}$ , where  $\Omega(b_i, r_j)$  is the initial amount attached by bidder  $b_i \in B$  to a resource  $r_j \in R$ .
2. The *weight* a package is defined as  $\Theta : B \times 2^R \rightarrow \mathbb{R}$ , where  $\Theta(b_i, S)$  is the weight for bidder  $b_i \in B$  of package  $S$ .
3. The *fair value* for a resource is defined as  $\Pi : B \times R \rightarrow \mathbb{R}$ , where  $\Pi(b_i, r_j) = \Theta(b_i, r_j) \times \Omega(b_i, r_j)$  for a bidder  $b_i \in B$  on resource  $r_j \in R$ .  
The fair value for a package is defined as  $\Pi : B \times 2^R \rightarrow \mathbb{R}$ , where  $\Pi(b_i, S)$  is the value obtained as  $\sum \Pi(b_i, r_j), \forall r_j \in S$ .
4. The *bid value* of a package is defined as  $\Upsilon : B \times 2^R \rightarrow \mathbb{R}$ , where  $\Upsilon(b_i, S)$  is the amount that the bidder  $b_i \in B$  is willing to give in exchange for the package  $S$ .
5. The *utility value* of a package is defined as  $\Gamma : B \times 2^R \rightarrow \mathbb{R}$ , where  $\Gamma(b_i, S) = \Upsilon(b_i, S) - \Pi(b_i, S)$ .
6. The *package cost* is defined as  $\Psi : B \times 2^R \rightarrow \mathbb{R}$ , where  $\Psi(b_i, S)$  gives the final winning amount for bidder  $b_i$  on package  $S$  after the bidding has ended.

Assume that the auctioneer and each bidder all have fair values for each of the individual resources (say, in dollars) as shown in Table 1. Every bidding process will have a base value initially assigned to an item from where the bidding proceeds. The fair values by a bidder and an auctioneer for each resource represent their measures of its actual value, and depend on their weights and their initial values (Definition 3.2). Thus, a bidder is willing to consider a resource at his fair value. Similarly, the auctioneer is willing to sell a resource at *his* fair value. However, bid value may be higher or lower than fair value and hence result in higher or lower utility values (Definition 3.2) depending on the need of the resource. Fair value for a combination of resources in the fairness table can be calculated as the sum of the

Example Fairness Table			
	Resource $r_0$	Resource $r_1$	Resource $r_2$
Bidder $b_0$	5	8	8
Bidder $b_1$	10	2	8
Bidder $b_2$	10	5	10
Auctioneer $a$	8	10	15

Table 1: Fair valuations for each resource by all bidders and auctioneer

fair value for each of the resources in that combination (fair values are considered additive where as the bid values are combinatorial in nature and not additive).

From Table 1, we can see that bidder  $b_0$  values resource  $r_0$  at \$5,  $r_1$  at \$8 and  $r_2$  at \$8. This means that bidder  $b_0$  is willing to pay \$5 for  $r_0$ , \$8 for  $r_1$ , and \$8 also for  $r_2$ ;  $b_0$  believes that no loss is incurred by the auctioneer in this trade. The fair value for the subset  $\{r_0, r_2\}$  for the bidder  $b_0$  is calculated as the sum of his the fair values for  $r_0$  and  $r_2$ , i.e., \$5 + \$8 = \$13. Similarly, the fair value for a package is the sum of the fair values of the comprising sets (Definition 3.2), i.e., for a package  $\{r_0\}$ ,  $\{r_1, r_2\}$ , the fair value is the sum of the fair values of  $\{r_0\}$  and  $\{r_1, r_2\}$ .

A bidder participates in the bidding process by quoting his bid for the packages. Let the bids raised by the bidders for the individual resource and different combination of resources be as given in Table 2. It can be seen that the bids raised by each of the bidder for different sets of resources may or may not be equal to the fair value of the respective set of resources. This is because the combinations may be complimentary or substitutes.

A bidder is considered to make bid zero for any sets of resources he does not wish to procure.

Example Bid Table							
	$r_0$	$r_1$	$r_2$	$\{r_0, r_1\}$	$\{r_0, r_2\}$	$\{r_1, r_2\}$	$\{r_0, r_1, r_2\}$
Bidder $b_0$	0	10	5	10	20	15	50
Bidder $b_1$	10	5	10	30	0	0	50
Bidder $b_2$	10	0	15	20	30	15	30

Table 2: Bids raised by the bidders for different combination of resources

With this terminology we proceed to explain fairness in subsequent sections.

### 3.2 Basic Fairness

In many MAS, there occurs a need of allocating the resources in an equitable manner, i.e., each agent gets an equitable share of the resources. Such allocations leave the agents with a feeling that they have received a fair share (Brams & Taylor, 1996). For example, if we

consider a method that would leave two agents feeling as if they had received 60% of the good then we would call it equitable. If one felt to be favored and had received 80% while the other agent believed to have received 60% then it would not be equitable (Brams & Taylor, 1996). This is quite difficult to access and tends to be quite subjective in many cases. We give a mechanism where this applies only in case of a tie, hence we consider a divisible resource which does not lose its value upon division and divide it equitably among bidders in proportion to their assigned weights. Each agent has a set of allocations he deems fair. An allocation is then said to achieve basic fairness in resource allocation if all agents deem it fair.

Each bidder  $b_i$  wants to maximize his chances of procuring the resource and individual utility given by  $\Gamma(b_i, S)$  represents the satisfaction of obtaining the resource. The most simple approach is that the satisfaction of a bidder does not depend on other bidders' satisfactions. The representation below considers that a package  $S$  is divisible and can be divided equitably among  $n$  bidders in proportion to their utility values.

The resource can be divided equitably in the ratio:  $\Gamma(b_i, S) / \sum_{i=1}^n \{\Gamma(b_i, S)\}$ , where weights are set freely by agents.

**Definition 3.3.** If each bidder  $b_i$  has a utility for a package  $S$  given by  $\Gamma(b_i, S)$ , and the package  $S$  can be divided equitably among  $n$  bidders in the ratio  $\Gamma(b_i, S) / \sum_{i=1}^n \{\Gamma(b_i, S)\}$ , then basic fairness is said to be achieved.

**Example 3.4.** Consider there to be three bidders for the divisible package  $S$ . The bidders' bid values, fair values and utility values are shown in Table 3.

Utility and Weight Table			
	Bidder $b_0$	Bidder $b_1$	Bidder $b_2$
Bid Value	24	16	20
Fair Value	18	12	16
Utility Value	6	4	4

Table 3: Example to demonstrate basic fairness

The calculations of ratios are done as shown below.

For bidder  $b_0$ ,  $6/14 = 0.43$ .

For bidder  $b_1$ ,  $4/14 = 0.285$ .

For bidder  $b_2$ ,  $4/14 = 0.285$ .

If the winning amount is \$100 then it is divided in the ratio 0.43 : 0.285 : 0.285 to achieve basic fairness, i.e., bidder  $b_0$  has to pay \$43, bidder  $b_1$  has to pay \$28.50, bidder  $b_2$  has to pay \$28.50.

This method of equitable allocation ensures that all agents deem the allocation to be fair. Therefore, we say that every agent believes that the set of resources is divided fairly among all the agents. This concept of fairness is termed as basic fairness.



This kind of fairness is required in applications wherein fairness is the key issue, rather than the individual satisfactions of the self-interested agents. In such applications, it becomes necessary to divide a package in an equitable fashion so that every agent believes that it is receiving its fair share from the set of resources. Hence, we see that every agent enjoys material equality and this ensures basic fairness among them.

### 3.3 Extended Fairness

In order to ensure egalitarian social welfare (Chevalleyre et al., 2005), basic fairness is alone not sufficient. We also need to address envy-freeness (Brams, 2005). Envy-free allocations result in each agent being at least as happy with its share of the goods as it would be with any of the other agents shares despite the difference in some payments for identical goods (Brams & Taylor, 1996; Koboldt et al., 2003). Here, we need to ensure that the allocation is perceived by all agents to be a fair allocation.

In a MAS, every agent assigns a fair value to each resource that determines its estimate of the value of the resource in quantitative terms. The fair value attached to each resource can be expressed in monetary terms in most MAS. Here, the agent believes that the allocated resource is fair if he receives the entire allocation and the value is according to his fair estimate.

However, it is important to mention that the fair value attached to each resource by an agent does not necessarily reflect the bid value of the resource. An agent may hold a higher or lower bid value for a resource irrespective of the fair value attached to the resource. Rather, the fair value attached to a resource is an estimate of the actual value of the resource in the system as perceived by an agent in quantitative terms. It means that an agent is always willing to trade a resource at its fair value.

Let there be  $k$  bidders who bid for a package  $S$ . Let each bidder  $b_i$  and auctioneer  $a$  (who is here only as a facilitator for achieving the items' fair value) give their fair values in the fairness table (as in the example in Table 1), which is open for all to see at the *end* of the bidding process.

**Definition 3.5.** Let us define some of the terms used in our discussion.

1.  $C$  is defined as the winning amount after the bidding process for a package  $S$ .
2.  $\xi : a \times 2^R \rightarrow \mathbb{R}$  defines the fair value of the auctioneer  $a$  for a package  $S$  denoted by  $\xi(S)$ .
3.  $\Pi(b_i, S)$  is defined as the fair value of the bidder  $b_i$  for a package  $S$ .
4. The *profit* denoted by  $\Phi$  is defined as the net amount  $C$  above the fair value of the auctioneer  $\xi(S)$  given by the bidder  $b_i$  for the package  $S$  and is calculated as difference  $C - \xi(S)$ .
5. The function *distribute* is defined as the amount  $x : B \times 2^R \rightarrow \mathbb{R}$  to be given back to the losing bidders  $b_i$  who bid for the winning package  $S$ .

6. The value *reward* is defined as  $reward : B \times 2^R \rightarrow \mathbb{R}$ , where  $reward = \Phi - x$ .

Now let us define extended fairness in resource allocation.

**Definition 3.6.** An allocation is said to satisfy extended fairness, if when a winning bidder  $b_i$  is allocated a package  $S$ : (i) if  $\Upsilon(b_i, S) > \xi(S)$ , then a losing bidder  $b_j$  is rewarded  $\Phi \times \left( \frac{\Pi(b_j, S) - \xi(S)}{\xi(S)} \right)$ ; and (ii) if  $\Upsilon(b_i, S) \leq \xi(S)$ , then no one gets a reward.

Consider the following scenarios:

1. The auctioneer makes a profit more than his fair value assigned initially for that package  $S$ . He distributes the profit among the losing bidders in proportion to their fair values for that package  $S$  as follows:

Let  $C$  be the winning bid which is greater than the fair value of the auctioneer, i.e.,  $\xi(S)$ . Therefore,  $\Phi = C - \xi(S)$  by Definition 3.5. The profit to be distributed for each losing bidder  $i$  is calculated by:

$$distribute \left( \Phi \times \left( \frac{\Pi(b_i, S) - \xi(S)}{\xi(S)} \right) \right)$$

Now, the incentive for the winning bidder is  $reward = \Phi - distribute \left( \Phi \times \left( \frac{\Pi(b_i, S) - \xi(S)}{\xi(S)} \right) \right)$ . Thus, the auctioneer has obtained his fair value and hence considers this allocation as fair. All the bidders get amounts according to their fair values, which makes them envy free.

2. The auctioneer gets a winning bid  $C$  which is exactly the same as the fair value  $\xi(S)$  associated with the package  $S$ . Now the profit is zero. Therefore, the auctioneer has obtained his fair value and hence considers this allocation as fair. All the bidders, though did not get any reward consider this allocation as envy-free as auctioneer too did not make any profits more than his own fair value.
3. The auctioneer gets a winning bid  $C$  less than the fair value  $\xi(S)$  attached by him for the package  $S$ . In this case, we try to minimize his loss as follows:

If the fair value given by bidder  $\Pi(b_i, S) \geq \xi(S)$ , then bidder  $b_i$  pays  $\xi(S)$ . Thus, the auctioneer has no loss as he gets his fair value and the bidder too is envy free since he considers that paying his fair value as fair. The other bidders are still envy free since the amount paid by the winning bidder is more than he actually won in the bidding process.

If the fair value given by bidder  $\Pi(b_i, S) < \xi(S)$  and  $\Pi(b_i, S) \leq C$ , then the payment does not change and he pays  $C$ , else he pays  $\Pi(b_i, S)$ . Only in this case auctioneer fails to get his fair value and the bidder does not get the distributed profit amount. The allocation is still envy-free for all the bidders but not for the auctioneer. This can be avoided if both bidder and auctioneer remain truthful in their fair values.

**Example 3.7.** An example of such a system can be explained with a scenario of auctioning of a painting. The contending bidders express their fair values through their sealed bids that is submitted to the auctioneer, i.e., each contending bidder believes that his quotation fulfills the value expected by the auctioneer and he is a competitive contender for the painting. We assume here that all bids are truthful. An unbiased auctioneer selects the bid which is the maximum for revenue maximization and the painting is allotted to him. Here the auctioneer would distribute the profits among losing bidders when he gets back his fair value. This takes care of envy-freeness. Hence, the allocation is perceived to be fair by the winning bidder and by all other bidders as it is allocated to the most deserving among all the bidders. The auctioneer also perceives this to be fair since he will obtain the fair value for the resource. Thus all participants perceive the allocation to an agent to be fair irrespective of the fair values attached by them. Therefore, extended fairness is said to be achieved.

To make these notions of fairness mathematically precise, we need a framework where fairness is a required property in resource allocation. However, we also see that resource allocation deals with another key issue of optimality in various MAS. The best example of resource allocation framework where both optimality and fairness are the key issues is Combinatorial Auctioning Systems (CAS).

#### 4. Fairness in Combinatorial Auctioning Systems (CAS)

Combinatorial Auctioning Systems are a kind of MAS which comprise an auctioneer and a number of self-interested bidders. The auctioneer aims at allocating the available resources among the bidders who, in turn, bid for sets of resources to procure them in order to satisfy their needs. The bidders aim at procuring the resources at minimum value during the bidding process, while the auctioneer aims at maximizing the revenue generated by the allocation of these resources. Thus, CAS refers to a scenario where the bidders bid for the set of resources and the auctioneer allocates the same to the highest-bidding agent in order to maximize the revenue. Hence, we see that optimality is one of the key issues in CAS.

An algorithm of Sandholm is used here to attain optimal allocation of resources. Sandholm proposes various methods for winner determination in combinatorial auctions (Sandholm, 2002). The search methodology can be used to obtain optimal allocation of resources. We can represent the Table 2 as a Bid tree using an algorithm of Sandholm (Sandholm, 2002). We can also carry out some preprocessing steps to make the steps faster without compromising the optimality (Narahari & Dayama, 2005; Sandholm, 2002). Thus we can determine the winning bidders.

However, besides optimality, another key issue desired by some auctioning systems is fairness. To incorporate this significant property in this resource allocation procedure, we propose an algorithm which uses the concept of extended fairness for each agent with basic fairness in case of a tie and determines the final payment made by the winning bidders.

The algorithm that we describe is based upon a CAS that uses an algorithm of Sandholm for achieving optimality, and an incentive compatible mechanism called Generalized Vickrey

Auction (GVA) along with Algorithm 1 as the pricing mechanism that determines the payments to be given by the winning bidders.

The Generalized Vickrey Auction (GVA) has a payoff structure that is designed in a manner such that each winning agent gets a discount on its actual bid. This discount is called a Vickrey Discount, and is defined by (Narahari & Dayama, 2005) as the extent by which the total revenue to the seller is increased due to the presence of that winning bidder, i.e., the marginal contribution of the winning bidder to the total revenue. The GVA framework requires significant transfer payments from bidders to auctioneer hence a redistribution mechanism is required to reduce the cost of implementation (Guo & Conitzer, 2009; Cavallo, 2006). Hence, after we obtain winning bidders from the algorithm of Sandholm, the GVA mechanism can be applied to get Package Cost (Definition 3.2) and Algorithm 1 can be used for redistribution of payments back to the bidders to achieve fair allocation. We give mathematical formulations to show that both kinds of fairness can be achieved in CAS.

#### 4.1 Notion of Fairness in Combinatorial Auctions

1. Each bidder and the auctioneer define its fair values in the fairness table (Table 1) before the start of the bidding process. It is a sealed matrix and is unsealed at the end of bidding process.
2. An allocation tree is constructed at the end of the bidding process to determine the optimum allocation and the winning bidders (Sandholm, 2002). Information about all the bidders in a tie is not discarded using some pre-defined criteria.
3. Calculate the package cost  $\Psi(b_i, S_j)$  (Definition 3.2) denoted by  $P_{i,j}$  which is the final winning bid amount for bidder  $b_i$  on package  $S_j$  is obtained after applying GVA scheme.
4. The fair value of the package won by each bidder is calculated, and the value is denoted as  $Q_{i,j}$  for the bidder  $b_i$  who wins the package  $S_j$ .
5. The fair value of each package is calculated using the fairness table of the auctioneer and is denoted as  $Q_{a,j}$  for a package  $S_j$ .
6. The values of  $Q_{a,j}$  and  $P_{i,j}$  are compared to determine the final payment by the bidder which is considered fair.

Now, we propose an algorithm which satisfies extended fairness in all cases, except in case of a tie.

#### 4.2 Notations Used in Algorithm 1

- $b_i$  is an arbitrary bidder  $i$  who belongs to the set of bidders  $B$ .
- $S_j$  is the winning package with complementaries and substitutes included, which is a subset of set  $R$ .

- In general, the Fairness Table for bidder  $b_i$  and Auctioneer  $a$  is defined as shown in Table 4.

Fairness Table					
	Resource $r_0$	Resource $r_1$	Resource $r_2$	...	Resource $r_{m-1}$
Bidder $b_0$	$\Pi(b_0, r_0)$	$\Pi(b_0, r_1)$	$\Pi(b_0, r_2)$	...	$\Pi(b_0, r_{m-1})$
Bidder $b_1$	$\Pi(b_1, r_0)$	$\Pi(b_1, r_1)$	$\Pi(b_1, r_2)$	...	$\Pi(b_1, r_{m-1})$
.	.	.	.	...	.
.	.	.	.	...	.
.	.	.	.	...	.
Bidder $b_{n-1}$	$\Pi(b_{n-1}, r_0)$	$\Pi(b_{n-1}, r_1)$	$\Pi(b_{n-1}, r_2)$	...	$\Pi(b_{n-1}, r_{m-1})$
Auctioneer $a$	$\xi(r_0)$	$\xi(r_1)$	$\xi(r_2)$	...	$\xi(r_{m-1})$

Table 4: Fair values for each resource by all bidders and auctioneer

- The fair value function by a bidder  $b_i$  for a resource  $r_j$  is given by  $\Pi(b_i, r_j) = d$ , where  $d \in \mathbb{N}$ .
- $Q_{i,j}$  is the fair value of resource  $r_j$  by bidder  $b_i$  where  $r_j \in R$  and  $b_i \in B$ .
- $Q_{a,j}$  is the fair value of resource  $r_j$  by auctioneer  $a$  where  $r_j \in R$ . Here we consider only a single auctioneer.
- The package Cost  $P_{i,j}$  (Definition 3.2) for bidder  $b_i$  obtained from the GVA scheme is represented as  $\Psi(b_i, S_j)$ . (The package cost is a function of bid values on the bundles of resources)
- The pay function by a bidder  $b_i$  is represented as  $pay(c)$  is the final payment to be made to the auctioneer by the bidder  $b_i$  where  $c$  is the bid amount.
- $\Phi$  (Definition 3.5) is the net amount above the fair value distributed by the auctioneer  $a$  to the bidders for a package  $S_j$ .
- *distribute* is a function which calculates the amount to be given back to the bidders who bid for the winning package  $S_j$  (Definition 3.5).
- *loss* is the net amount below the fair value given by the auctioneer to the package  $S_j$ .

### 4.3 Flow of Algorithm 1

In Algorithm 1, we calculate the package cost and the fair values of bidder and auctioneer given in the lines 1–3. These are calculated in the beginning and are represented as  $P_{i,j}$ ,  $Q_{i,j}$  and  $Q_{a,j}$  respectively.

In lines 4–9, we have the first **if** condition where the package cost  $P_{i,j}$  is greater than the fair value assigned by the auctioneer for the package  $Q_{a,j}$ . If this evaluates to TRUE, then the bidder pays the amount but the net profit calculated is distributed among all the

bidders who bid for that package proportional to their bids.

In lines 10–12, we have the second **if** condition where the package cost  $P_{i,j}$  is equal to the fair value assigned by the auctioneer for the package  $Q_{a,j}$ . If this evaluates to TRUE, since there is no profit the bidder still pays and there is no amount distributed to the winning package bidders.

In lines 13–32, we have the third ‘if’ condition where the package cost  $P_{i,j}$  is less than the fair value assigned by the auctioneer for the package  $Q_{a,j}$ . If this evaluates to TRUE, there is a loss for the auctioneer so we try to minimize the loss by checking the additional cases as follows.

First at line 15, if the fair value of bidder  $Q_{i,j}$  is greater than fair value of auctioneer  $Q_{a,j}$  evaluates to TRUE, then the bidder will have to pay only  $Q_{a,j}$ . This prevents loss for auctioneer and also the bidder deems it as fair.

Secondly at line 19, if the fair value of bidder  $Q_{i,j}$  is equal to the fair value of auctioneer  $Q_{a,j}$  evaluates to TRUE, then the bidder will have to pay only  $Q_{a,j}$  as in the previous condition. Similar to the previous condition this prevents loss for auctioneer and also the bidder deems it as fair.

Finally at line 23, if the fair value of bidder  $Q_{i,j}$  is less than fair value of auctioneer  $Q_{a,j}$  evaluates to TRUE, then we have to see the additional two conditions as follows.

If the fair value of bidder  $Q_{i,j}$  is less than or equal to package cost of bidder  $P_{i,j}$  then the bidders’ final payment remains the same, i.e.,  $P_{i,j}$ .

If the fair value of bidder  $Q_{i,j}$  is greater than the package cost of bidder  $P_{i,j}$  then the bidders’ final payment is  $Q_{i,j}$ . These are presented in Algorithm 1.

#### 4.4 Algorithm to Incorporate Extended Fairness

**Data:** package cost, fair value of winning bidder, fair value of auctioneer

**Result:** Final Payment by the bidder

```

1  $P_{i,j} \leftarrow \Psi(b_i, S_j)$                                 /* where  $P_{i,j} \in \mathbb{R}$  */
2  $Q_{i,j} \leftarrow \Pi(b_i, S_j)$                             /* where  $Q_{i,j} \in \mathbb{R}$  */
3  $Q_{a,j} \leftarrow \xi(S_j)$                                 /* where  $Q_{a,j} \in \mathbb{R}$  */
4 if  $P_{i,j} > Q_{a,j}$  then
5    $pay(P_{i,j})$ ;
6    $\Phi \leftarrow (P_{i,j} - Q_{a,j})$ ;
7    $distribute(\Phi \times [(Q_{k,j} - Q_{a,j})/Q_{a,j}])$ ; /* among other bidders who bid for
   package  $S_j$  */
8
9 end
10 if  $P_{i,j} = Q_{a,j}$  then
11    $pay(P_{i,j})$ ;
12 end
13 if  $P_{i,j} < Q_{a,j}$  then
14    $loss \leftarrow (Q_{a,j} - P_{i,j})$ ; /* Auctioneer can recover as follows */
15   if  $Q_{i,j} > Q_{a,j}$  then
16      $pay(Q_{a,j})$ ; /* Bidder's estimate of fair value is more than  $P_{i,j}$  */
17   end
18   if  $Q_{i,j} = Q_{a,j}$  then
19      $pay(Q_{a,j})$ ; /* fair value is same and  $Q_{i,j}$  greater than  $P_{i,j}$  */
20   end
21   if  $Q_{i,j} < Q_{a,j}$  then
22     if  $Q_{i,j} \leq P_{i,j}$  then
23        $pay(P_{i,j})$ ; /* Bidder's final payment remains the same */
24     else
25        $pay(Q_{i,j})$ ; /* Bidder's final payment is  $Q_{i,j}$  */
26     end
27   end
28 end
29 end
30 end
31 end
32 end

```

**Algorithm 1:** Algorithm incorporating extended fairness

#### 4.5 Handling a Case of a Tie—Incorporating Basic Fairness

Unlike traditional algorithms, we do not discard the bids in the case of a tie on the basis of some pre-decided criterion. We consider these cases in our algorithm to provide basic fairness to the bidders. In case of a tie, we shall measure the utility value of the resource to

each bidder in the tie. The utility value of a resource to a bidder is the quantified measure of satisfaction or happiness derived by the procurement of the resource (Definition 3.2).

The bidders maximize this utility value to quantify the importance and their need for the resource. Thus, the higher the utility value, the greater is the need for the package. In such a case, fairness can be achieved if the package  $S$  is divided among all the bidders in a proportional manner, i.e., in accordance to the utility value attached to the package by each bidder.

**Example 4.1.** Let us consider the same example to explain the concept of basic fairness in our system. From Table 2, we observe that the optimum allocation attained through allocation tree comprises the package  $r_0, r_1, r_2$  as it generates the maximum revenue of \$50. However, we see that this bid is submitted by the two bidders,  $b_0$  and  $b_1$ .

Thus, we calculate the fair value of the package  $S_j = \{r_0, r_1, r_2\}$  for the bidder  $b_0$  and  $b_1$ , i.e.,  $\Pi(b_0, S_j) = 5 + 8 + 8 = \$21$  and  $\Pi(b_1, S_j) = 10 + 2 + 8 = \$20$ . Thus, the utility value of the package  $S$  for the bidder  $b_0$  and  $b_1$  is as follows:

For bidder  $b_0$ ,  $\Gamma(b_0, S_j) = 50 - 21 = \$29$ , and

For bidder  $b_1$ ,  $\Gamma(b_1, S_j) = 50 - 20 = \$30$ .

Hence, the package  $S$  is divided among bidders  $b_0$  and  $b_1$ , in the ratio of 29 : 30. In other words, bidder  $b_0$  gets 49.15% and bidder  $b_1$  gets 50.85% of the package  $S_j$ .

The payment made by the bidders is also done in the similar proportional manner similar to Example 3.4.

The bidders  $b_0$  and  $b_1$  make their respective payments in the ratio of 29 : 30 to make up a total of \$50 for the auctioneer, i.e., bidder  $b_0$  pays \$24.65 and bidder  $b_1$  pays \$25.35 to the auctioneer for their respective shares.

Hence, we see that extended fairness as well as basic fairness are achieved in a CAS using our approach. We take into account the fair estimates of the auctioneer and the bidders for each resource to ensure that fairness is achieved to auctioneer as well as the bidders. A detailed analysis of our mechanism is in the following section.

## 5. Analysis

Using the solution concept of dominant strategies and mechanism design with quasilinear preferences, we can analyze the following.

We say that the agents' preferences are quasilinear when they satisfy the conditions given below: first we are in a setting where the mechanism can choose to charge or reward an agent an arbitrary amount. Second, and more restrictive, is that an agent's utility of a choice cannot depend on the money that he has, i.e., his value is the same whether he is rich or poor. Finally, the agents care only about the choice selected and their own payments, i.e., they are not concerned about monetary payments made or received by other agents.



## 5.1 Fairness

We say that extended fairness is achieved when a bidder procures a resource for an amount that is equal to his estimate of fair value of that resource. In such a case, the bidder believes that the resource was procured by it at a fair amount irrespective of other bidders estimate of fair value of that resource. This is according to the last condition of quasilinear preference. Thus, the allocation is believed to be extendedly fair as per the estimates of the winning bidder.

We also see that basic fairness is achieved in our system when there is more than one bidder who has raised equal bid for the same set of resources. In such a case, we divide the set of resources among all the bidders so as to ensure fairness to all the bidders in a tie. However, this division of resources set is done in a proportional manner. We intend to divide the resource such that the bidder holding highest utility value to it should get the biggest share. To ensure this, we calculate the utility value (i.e.,  $\Gamma(b_i, S_j) = \Upsilon(b_i, S_j) - \Pi(b_i, S_j)$ ) of the set of resources to each bidder and divide the set in the ratio of these values among the respective bidders. Thus, we see that each bidder procures his basic share of the set of resources in accordance to the basic importance attached by the bidder to the set of resources. Due to the achievement of fairness through our payment scheme, the bidders are expected to show willingness to participate in the auctions.

## 5.2 Higher Rewards

Here we show that a bidder is encouraged to bid higher as he gets rewards proportional to his bids which are fair.

**Theorem 5.1.** Given a CAS, a bidder has an incentive to bid higher using the extended fairness algorithm 1 as he gets higher rewards which are fair provided:

1. the bid value  $P_{i,j}$  is always greater than or equal to fair value  $Q_{a,j}$  on package  $S_j$ .
2.  $Q_{i,j}$  of winning bidder is always greater than or equal to  $Q_{i,j}$  of the losing bidder.

*Proof.* Assume any two bidders  $b_x, b_y \in B$  who bid for package  $S_j$ . Assume values:  $P_{x,j}, Q_{x,j}$  for bidder  $b_x$ .  $P_{y,j}, Q_{y,j}$  for bidder  $b_y$ .  $Q_{a,j}$  for auctioneer  $a$ .

The first condition is  $P_{x,j} > Q_{a,j}$  and  $Q_{x,j} > Q_{y,j}$  hence we have a profit  $\Phi_x$  which is distributed in the ratio  $\Phi_x \times \left(\frac{Q_{x,j}-Q_{a,j}}{Q_{a,j}}\right)$  to  $b_x$  and  $\Phi_x \times \left(\frac{Q_{y,j}-Q_{a,j}}{Q_{a,j}}\right)$  to  $b_y$ . This gives proportional as well as fair incentives to  $b_x$  and  $b_y$ . This also gives a higher reward to  $b_x$  since  $Q_{x,j} > Q_{y,j}$ .

For notational convinience, let  $k$  represent  $\frac{Q_{x,j}-Q_{a,j}}{Q_{a,j}}$  and  $l$  represent  $\frac{Q_{y,j}-Q_{a,j}}{Q_{a,j}}$ .

The second condition is  $P_{x,j} > Q_{a,j}$  and  $Q_{x,j} > Q_{y,j}$  hence we have a profit  $\Phi_y$  which is distributed as  $\Phi_y \times k$  and  $\Phi_y \times l$  where  $k, l$  are constant for  $b_x$  and  $b_y$ . When  $\Phi_y > \Phi_x$  we see a greater amount of reward is given to higher bidding and hence bidders are encouraged to bid more.  $\square$

**Theorem 5.2.** Given a CAS, a bidder gets higher rewards using the extended fairness algorithm 1 if his fair value  $Q_{i,j}$  satisfies the condition  $Q_{a,j} > Q_{i,j} > 2 \times Q_{a,j}$  for a package  $S_j$ .

*Proof.* Assume a bidder  $b_i \in B$  who bids for package  $S_j$  and wins it. Assume values:  $P_{i,j}, Q_{i,j}$  for bidder  $b_i$ .  $Q_{a,j}$  for auctioneer  $a$ .

The first condition is when the winning bidder  $b_i$  has given a low fair value for a package  $S_j$  intentionally, i.e.,  $Q_{i,j} < Q_{a,j}$ . Now, his ratio is calculated as  $k = \frac{Q_{i,j} - Q_{a,j}}{Q_{a,j}}$  which is negative. He has to distribute an extra amount of the same proportion, i.e.,  $distribute\left(\Phi \times \left(2 \times \frac{Q_{i,j} - Q_{a,j}}{Q_{a,j}}\right)\right)$ . Hence,  $reward = \Phi - distribute\left(\Phi \times \left(2 \times \frac{Q_{i,j} - Q_{a,j}}{Q_{a,j}}\right)\right)$ . Therefore, bidder has to pay  $P_{i,j} - reward$ .

The second condition is when the winning bidder  $b_i$  has given a very high fair value for a package  $S_j$ , i.e.,  $Q_{i,j} > 2 \times Q_{a,j}$ . Now, his ratio is calculated as  $k = \frac{Q_{i,j} - Q_{a,j}}{Q_{a,j}}$  which is greater than 1. He has to distribute  $\Phi \times \left(\frac{Q_{i,j} - Q_{a,j}}{Q_{a,j}}\right)$  which is greater than  $\Phi$ . Therefore, the extra amount to be distributed would be added to  $P_{i,j}$  and hence ends up paying higher amount without any rewards.

The third condition is when the winning bidder  $b_i$  has given a true fair value for a package  $S_j$ , i.e.,  $Q_{a,j} > Q_{i,j} > 2 \times Q_{a,j}$ . Now, his ratio is calculated as  $k = \frac{Q_{i,j} - Q_{a,j}}{Q_{a,j}}$  which is a proper fraction. Hence,  $reward = \Phi - distribute\left(\Phi \times \left(\frac{Q_{i,j} - Q_{a,j}}{Q_{a,j}}\right)\right)$ . Therefore, bidder  $b_i$  has to pay  $P_{i,j} - reward$  from definition 3.5.

Clearly, we can see that the maximum reward is possible only in the third condition, where the fair value is neither too low nor very high. Thus, Algorithm 1 provides higher rewards if fair value for a package  $S_j$  satisfies the condition  $Q_{a,j} > Q_{i,j} > 2 \times Q_{a,j}$ .  $\square$

### 5.3 Other Issues

Now let us discuss some issues considering the quasilinear mechanism.

#### 5.3.1 TRUTHFULNESS

Consider the following definition by (Shoham & Leyton-Brown, 2009).

**Definition 5.3.** A quasilinear mechanism is *truthful* if it is direct and  $\forall i$ , bidder  $b_i$ 's equilibrium strategy is  $\Upsilon(b_i, S_j) = \Pi(b_i, S_j)$ .

(Note that (Shoham & Leyton-Brown, 2009) uses  $v_i$  for what we denote by  $\Pi(b_i, S_j)$  and  $\hat{v}_i$  for what we denote by  $\Upsilon(b_i, S_j)$ .)

**Theorem 5.4.** In our mechanism, the bidder  $b_i$ 's equilibrium strategy is  $\Upsilon(b_i, S_j) = \Pi(b_i, S_j)$  so it is truthful.

*Proof.* Assume a bidder  $b_i \in B$  who bids for package  $S_j$  provides his fair value  $\Pi(b_i, S_j)$  in the fairness table. Let us denote the strategy chosen by  $b_i$ . i.e.,  $\Upsilon(b_i, S_j) = \Pi(b_i, S_j)$  to be  $d_i$  which is a dominant strategy as per our assumption (Line 10 in Algorithm 1 and Theorem 5.2).

Assume that the bidder  $b_i$  would be better off declaring a fair value  $\Pi(b_i, S_j)'$  instead of  $\Pi(b_i, S_j)$  to our mechanism. This implies that  $b_i$  has chosen a different strategy  $d_i'$  instead of  $d_i$  which is not in equilibrium, contradicting our assumption that  $d_i$  is the dominant strategy for  $b_i$ .  $\square$

This means that here the only action available to an agent is to reveal his private information. Any solution to a mechanism design problem can be converted into one in which agents always reveal their true preferences, if the new mechanism “lies for the agents” in just the way they would have chosen to lie to the original mechanism. Thus the new mechanism is dominant-strategy truthful (Shoham & Leyton-Brown, 2009).

In our algorithm the bidder or auctioneer benefit only when they give their fair value truthfully as in cases where  $P_{i,j} > Q_{a,j}$  and  $P_{i,j} = Q_{a,j}$ , where he gets the incentives as profits are distributed. But if the fair value is not truthful then he risks going to the case  $P_{i,j} < Q_{a,j}$  and  $Q_{i,j} < Q_{a,j}$  where naturally he is denied of any benefits. Thus if he lies to the mechanism to gain profits he would not succeed as he would have chosen a strategy which leads to loss.

### 5.3.2 EFFICIENCY

Consider the efficiency with respect to the package won by the bidder  $b_i$  denoted by  $S_j$ . We define  $S_j'$  as a subset of resources which are not won by the bidder  $b_i$ .

Consider the following definition by (Shoham & Leyton-Brown, 2009).

**Definition 5.5.** A quasilinear mechanism is *strictly Pareto efficient*, or just *efficient*, if in equilibrium it selects a choice  $S_j$  such that  $\sum_i \Pi(S_j) \geq \sum_i \Pi(S_j')$

(Note that (Shoham & Leyton-Brown, 2009) uses  $x$  for what we denote by  $S_j$ .)

**Theorem 5.6.** In our mechanism, the bidder  $b_i$ ’s equilibrium strategy is to select choice  $S_j$  such that  $\sum_i \Pi(S_j) \geq \sum_i \Pi(S_j')$  so it is efficient.

*Proof.* Assume a bidder  $b_i \in B$  has chosen a dominant strategy  $d_i$  selects a choice  $S_j$  such that  $\sum_{b_i} \Pi(b_i, S_j) < \sum_{b_i} \Pi(b_i, S_j')$ . This implies that sum of fair values of items in selected package  $S_j$ , is not more efficient than the sum of items not in package  $S_j$ . Thus there is another strategy  $d_i'$  which selects a choice  $S_j'$  which is more efficient than  $S_j$  (Theorem 5.1). Hence,  $d_i$  was not in equilibrium as  $d_i'$  is the dominant strategy. This is a contradiction to our assumption that  $d_i$  was the dominant strategy.  $\square$

An efficient mechanism selects the choice which maximizes the sum of the agents’ utilities, disregarding the the monetary payments they are required to pay. This can be shown

in our algorithm concept where the choice is made on the agents' fair values which helps in maximizing its profits. Thus, the efficiency is defined in terms of the true fair values and not the declared value in the bid table (Table 2).

### 5.3.3 INCENTIVE COMPATIBILITY

The combinatorial auction can be made incentive compatible using the Generalized Vickrey Auction (GVA) and Algorithm 1. The payment using GVA can be explained by assuming that all agents follow their dominant strategies and declare their values truthfully. Each agent is made to pay his social cost; the aggregate impact that his participation has on other agents utilities (Shoham & Leyton-Brown, 2009).

The payment mechanism described in our system is incentive compatible, i.e., they fare best when they reveal their private information truthfully in certain cases. As shown in Theorem 5.1 and Theorem 5.2 the bidders following dominant strategies in Algorithm 1 is bound to get higher incentives.

Thus the GVA and Algorithm 1 enables our mechanism to be incentive compatible.

### 5.3.4 OPTIMALITY

Optimality is a significant property that is desired in a CAS. We ensure this property by the use of an algorithm of Sandholm (Sandholm, 2002) in our system. It is used to obtain the optimum allocation of resources so as to maximize the revenue generated for the auctioneer. Thus, the output obtained is the most optimal output and there is no other allocation that generates more revenues than the current allocation.

## 6. Conclusion

We have shown that fairness can be incorporated in CAS from our methodology. Extended fairness as well as basic fairness can be attained through our payment mechanism. Optimal allocation is obtained through an algorithm of Sandholm, and the other significant properties like allocative efficiency and incentive compatibility are also achieved. This is an improvement because in the existing world of multi-agent systems, there do not seem to be many studies that attempt to incorporate optimality as well as fairness. The present paper addresses this lack in a specific multi-agent system, namely, the CAS.

The Nigerian Communications Commission (NCC) faced problems in giving incentives to bidders who divulge their preferences and bidders were not keen on divulging it since it may lead to more adverse competition. Our algorithm for extended fairness takes care of this problem as bidders receive more incentives with higher bids. Since the preferences given by the bidders in the fairness table is confidential and sealed, they need not worry about their preferences being disclosed to competitors.

The framework described can also be extended in several ways: first is to de-centralize the suggested algorithm, to avoid use of a single dedicated auctioneer. Especially in distributed computing environments, it would be best for there to be a method to implement

the suggested algorithm (or something close to it) without requiring an agent to act as a dedicated auctioneer (Rasheedi et al., 2009).

A second important extension would be to find applications for the work. Some applications that suggest themselves include distribution of land (a matter of great concern for governments and people the world over) in a fair manner. In land auctions where a tie occurs, no pre-defined or idiosyncratic method need be used to break the tie; rather, the allocation can be done fairly in the manner suggested.

A third important extension is to experiment with the grid computing framework (Bapna et al., 2008). The applicability of fairness scheme in grid computing while allocating resources and its impact on the expected revenue would be an interesting application area.

Fairness is also an important and pressing concern in the computing sciences and information technology, particularly, in distributed computing (Lamport, 2000). It is therefore also of interest to see how our method for achieving fairness could be applied in such contexts.

## References

- Bapna, R., Das, S., Garfinkel, R., & Stallaert, J. (2008). A market design for grid computing. *INFORMS JOURNAL ON COMPUTING*, 20, 100–111.
- Barbanel, J. B., & Brams, S. J. (2004). Cake-division with minimal cuts: Envy-free procedures for three persons, four persons, and beyond. *Mathematical Social Sciences*, 48, 251–270.
- Biggart, N. W. (2002). *Readings in economic sociology*. Wiley-Blackwell.
- Brams, S. J. (2005). *Fair Division*. Oxford University Press.
- Brams, S. J., & Taylor, A. D. (1996). *Fair Division*. Cambridge University Press.
- Bredin, J., Maheswaran, R. T., Imer, C., Basar, T., Kotz, D., & Rus, D. (2000). A game-theoretic formulation of multi-agent resource allocation. In Sierra, C., Gini, M., & Rosenschein, J. S. (Eds.), *Proceedings of the Fourth International Conference on Autonomous Agents*, pp. 349–356, Barcelona, Catalonia, Spain. ACM Press.
- Cavallo, R. (2006). Optimal decision-making with minimal waste: Strategyproof redistribution of vcg payments. In *Proc. of the 5th Int. Joint Conf. on Autonomous Agents and Multi Agent Systems (AAMAS'06)*, Hakodate, Japan.
- Chevaleyre, Y., Dunne, P. E., Endriss, U., Lang, J., Lemaitre, M., Maudet, N., Padget, J., Phelps, S., Rodriguez-Aguilar, J. A., & Sousa, P. (2006). Issues in multiagent resource allocation. *Informatica*, 30, 3–31.
- Chevaleyre, Y., Endriss, U., Estivie, S., & Maudet, N. (2005). Welfare engineering in practice: On the variety of multiagent resource allocation problems. *Engineering Societies in the Agents World*, V, 335–347.
- Conitzer, V. (2009). *Auction protocols*. Submitted for publication. To appear as chapter in CRC Algorithms and Theory of Computation Handbook.

- Foley, D. K. (1967). Resource allocation and the public sector. *Yale Economics Studies*, 7, 45–98.
- Guo, M., & Conitzer, V. (2009). Worst-case optimal redistribution of vcg payments in multi-unit auctions. *Games and Economic Behavior*, 67(1), 69–98.
- Koboldt, C., Maldoom, D., & Marsden, R. (2003). The first combinatorial spectrum auction: Lessons from the nigerian auction of fixed wireless access licences. In *ITS 14th European Regional Conference*, pp. 1–35, Dotecon Ltd, London, UK. DotEcon.
- Lahaie, S., & Parkes, D. C. (2009). Fair package assignment. In *Proc. First Conf. on Auctions, Market Mechanisms and Their Applications (AMMA 2009)*.
- Lamport, L. (2000). Fairness and hyperfairness. *Distributed Computing*, 13(4), 239–245.
- Liao, G.-Y., & Hwang, J.-J. (2001). A trustworthy internet auction model with verifiable fairness. *Internet Research*, 159–166.
- Moulin, H. (2004). *Fair Division and Collective Welfare*. Cambridge, MA:MIT Press.
- Narahari, Y., & Dayama, P. (2005). Combinatorial auctions for electronic business. *Sadhna*, 30, 179–211.
- Nisan, N. (2000). Bidding and allocation in combinatorial auctions. In *EC '00: Proceedings of the 2nd ACM Conference on Electronic Commerce*, pp. 1–12, New York, NY, USA. ACM.
- Rabin, M. (1993). Incorporating fairness into game theory and economics. *American Economic Review*, 83, 1281–1302.
- Rasheedi, L., Chatterjee, N., & Rao, S. (2009). Fair resource allocation in distributed combinatorial auctioning systems. In Garg, V., Wattenhofer, R., & Kothapalli, K. (Eds.), *Distributed Computing and Networking: Proceedings of the 10th International Conference on Distributed Computing and Networking (ICDCN 2009)*, Springer LNCS 5408, pp. 212–217.
- Sandholm, T. (2002). Algorithm for optimal winner determination in combinatorial auctions. *Artificial Intelligence*, 135, 1–54.
- Shapley, L. S. (1953). A value of n-person games. *Contributions to the Theory of Games*, II, 307–317.
- Shoham, Y., & Leyton-Brown, K. (2009). *MULTIAGENT SYSTEMS Algorithmic: Game-Theoretic, and Logical Foundations*. Cambridge University Press.
- Sycara, K. (1998). Multi-agent systems. *AI Magazine*, 10(2), 79–93.